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STUDY OF EPICYCLIC GEAR TRAINS USING GRAPH THEORY

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ABSTRACT

In this paper, graph theory is used for the study of epicyclic gear trains or planet gear trains (E.G.T. or P.G.T.). A PGT has been represented by its functional schematic, structural and kinematic graph. Advantages of kinematic graph representation of E.G.T. have been listed. Structural analysis of P.G.T. has been carried out.

KEYWORDS: Epicyclic Gear Train, Kinematic Graph, Mobility.

INTRODUCTION

Gear trains are used to transmit motion and / or power from one rotating shaft to another. A historical review of gear trains, from 3000 B.C. to the 1960s, can be found in Dudley [1]. A gear train is called an *ordinary gear train* if all the rotating shafts are mounted on a common stationary frame and a planetary gear train (PGT if some gears not only rotate about their own joint axes, but also revolve around some other gears.

NOTATIONS USED

The following notations have been used.
E.G.T.: Epicyclic Gear Train
ci : degrees of constraint on relative motion imposed by joint *i*.
F or dof: degrees of freedom of a mechanism.
fi : degrees of relative motion permitted by joint *i*.
j : number of joints in a mechanism, assuming that all joints are binary.
ji: number of joints with *i* dof; namely, *j*1 denotes the number of 1-dof joints, *j*2 denotes the number of 2-dof joints, and so on.
L: number of independent loops in a mechanism.
n: number of links in a mechanism, including the fixed link.
λ: degrees of freedom of the space in which a mechanism is intended to function.
S: spherical kinematic pair (dof =3).
E: plane kinematic pair (dof =3).
G: gear pair (dof =2).
j_g: no. of gear pair (dof =2).
j_t :no.of turning (revolute) pair.
fp: number of passive dof in a mechanism
λ is called motion parameter .λ = 6 (for spatial mechanisms kinematic chain) and λ = 3 (for planar and spherical mechanism kinematic chain).
di: no. of joints on link i.

FUNCTIONAL SCHEMATIC REPRESENTATION

For clarity and simplicity, only those functional elements that are necessary to the structural topology of a mechanism are shown [see Figure-3(a)].Two E.G.T. with internal and external gear mesh may have identical structural topology as shown in see Figure-4.

STRUCTURAL REPRESENTATION

Each link of a mechanism is represented by a polygon whose vertices represent the kinematic pairs. A binary, ternary, quaternary, ----links are represented by a line with two end vertices, across-hatched triangle with three vertices, a quaternary link with cross-hatched quadrilateral with four vertices, and so on.Figure-6 shows the structural representation of links. The plain and solid vertices of links denote revolute pairs and gear pairs respectively.Figure-3 illustrates the side view of the P.G.T. shown in Figure-2.Figure-6 shows the schematics and structural representations of link assortments used in geared kinematic chains.

KINEMATIC GRAPH REPRESENTATION

In a kinematic graph representation, the vertices denote links and the edges denote joints of a mechanism. The edge connection between vertices corresponds to the pair between links. To differentiate between various pairs, the edges can be labeled or colored. For example, the gear pairs and turning pair in a gear train are represented by thick edges and thin edges respectively.Figure-5 illustrates the graph representation of the P.G.T. shown inFigure-2.Figure-4 represents a gear set with two external gear and internal meshes.Figure-7 shows three different kinematic representations of four E.G.Ts.

Advantages of Using Graph Representation

The advantages of using the graph representation are:

1. Euler's equation can be applied to obtain the *loop mobility criterion* of mechanisms directly.
2. The similarity and difference between two different mechanisms can be easily identified.
3. Graphs may be used as an aid for computer-aided kinematic and dynamic analysis of mechanisms. For example, Freudenstein and Yang [2] applied the theory of fundamental circuits for the kinematic and static force analysis of planar spur gear trains. The theory was subsequently extended to the kinematic analysis of bevel-gear robotic mechanisms [3]. Recently, a systematic methodology for the dynamic analysis of gear coupled robotic mechanisms was developed [4].
4. Graph theory may be used for systematic enumeration of mechanisms.[5,6,7,8,9,10,11,12,13].
5. Graphs can be used for systematic classification of mechanisms. A single collection of graphs can be used to generate a number of mechanisms [14, 8, 16].
6. Graphs may use as help in automated sketching of mechanisms [17].

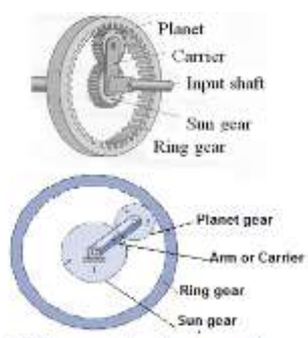
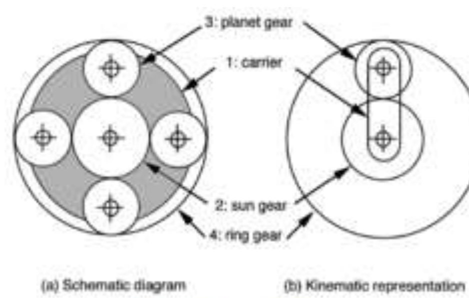
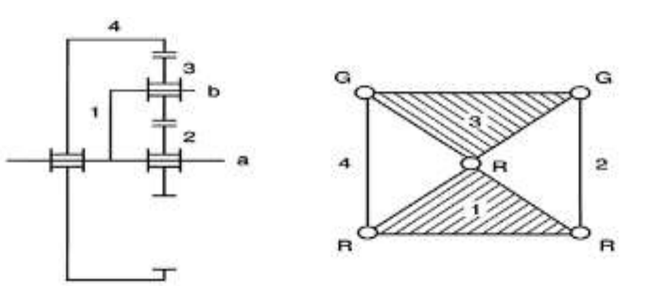


Fig-1: Components of a planet gear train



(a) Schematic diagram (b) Kinematic representation

Figure-2: Schematic diagram and kinematic representation of E.G.T.



(a) Functional schematic (b) Kinematic structural representation

Figure-3: Functional and structural representations of the P.G.T. shown in Figure-2

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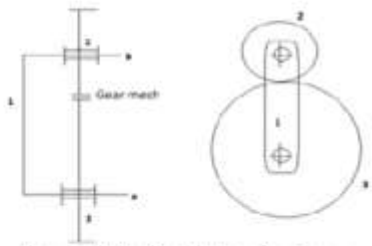

Figure-4(a): Functional schematic of spur gear with an external gear mesh

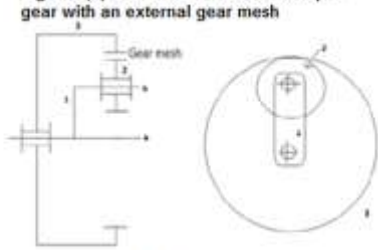

Figure-4(b): Functional schematic of a spur gear with an internal mesh.

Figure-4: Functional schematics of two gear sets that share the same structural topology

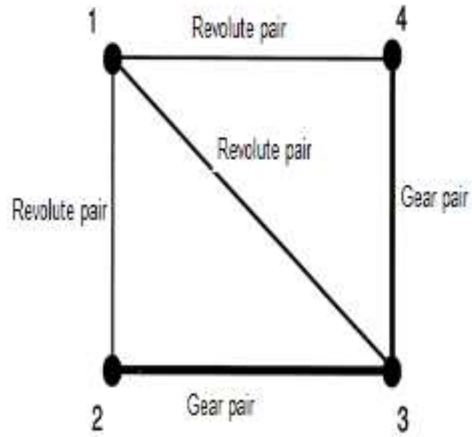

Figure-5: Kinematic graph representation of the P.G.T. shown in Figure-2.

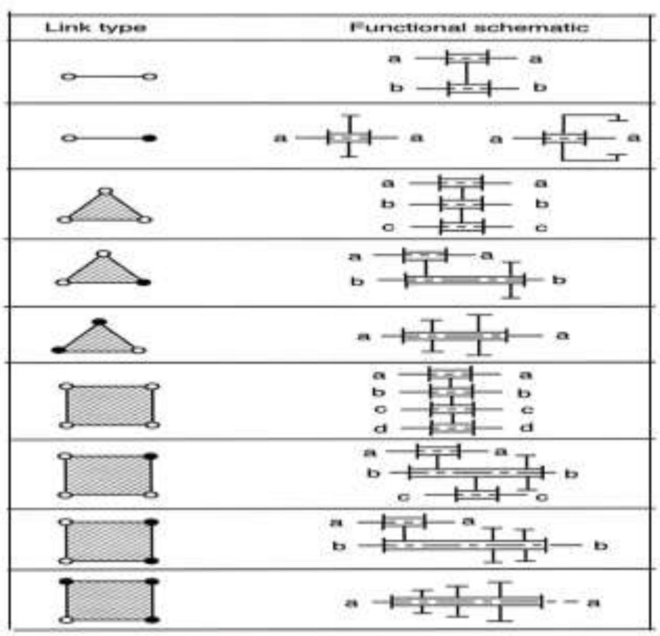

Figure-6: Link assortment used in gear kinematic chain

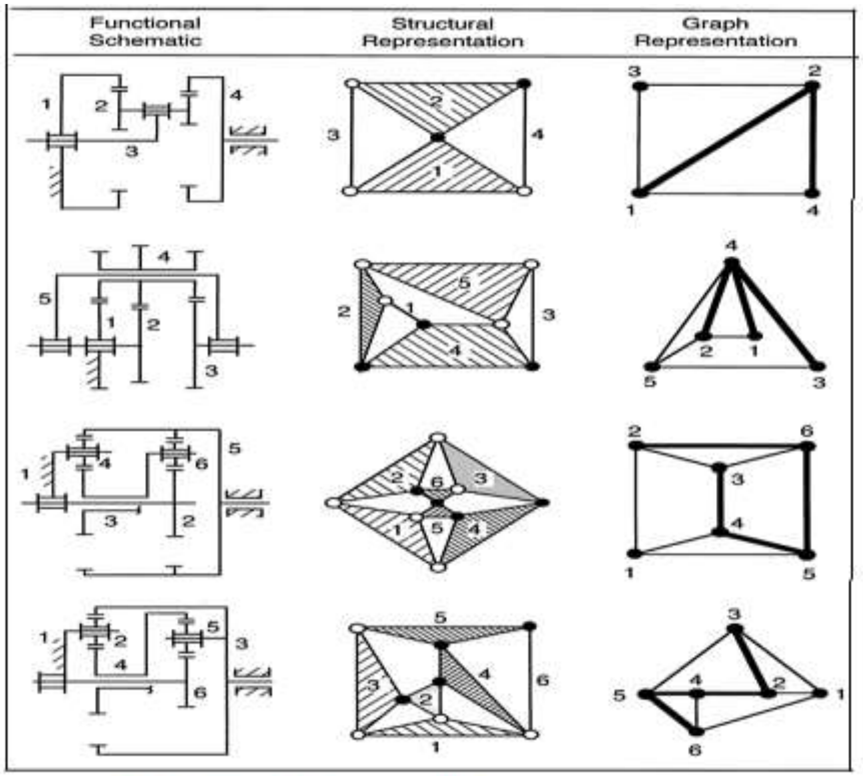

Figure-7: Kinematic representation of 4 P.G.T.

STRUCTURAL CHARACTERISTICS

There are revolute (R), prismatic (P) and gear (G) pairs in the planar geared mechanisms. If j_i denote the number of i-dof joints are connected by then total number of joints are given by eq.(1).

$$j=j1+j2 \text{ ----------------- } (1)$$

Because the revolute and prismatic pairs have one-dof and the gear pair two-dof. So, the sum of the dof in all pairs is given by eq.(2).

$$\sum_{i=1}^{j}(fi)=j1+2j2. \text{ ----------- } (2)$$

Subtracting eq.(1) from (2), we get eq.(3).

$$\sum_{i=1}^{j}(fi)= j2. \text{ --------------- } (3)$$

We also know that

$$F = \lambda\,(n - j - 1) + \sum_{i=0}^{j} fi \text{ --------- } (4)$$

Eq.(4) is called Grübler or Kutzbach criterion.Eq. (4)can be represented by introducing the term fp.

$$F = \lambda\,(n - j - 1) + \sum_{i=1}^{j}(fi - fp) \text{ --------- } (5)$$

In general,

F >0 , the mechanism has F dof, F = 0, the mechanism becomes a structure with zero dof, F <0, the mechanism becomes an over-constrained structure.

An equation among L, n and j in a KC can be developed. For a single-loop mechanisms KC (planar, spherical, or spatial), n = j , and all the links are binary. By increasing a mechanism KC from 1 to L loops, the difference between the j and n is increased by L − 1. So,

$$L = j - n + 1 \text{ ---------------} (6)$$

Eq. (6) is Euler's equation. From eq. (6) and (4), we get

$$\sum_{i=0}^{j} fi = F + \lambda L \text{ --------------- } (7)$$

Eq.(7) is known as the loop mobility criterion. Substituting λ=3 into eq.(7), we get

$$j+j2=F+3L \text{ . ----------------- } (8)$$

Hence, the no. of joints and of links in a geared mechanism depends on the gear pairs apart from the dof and L. It has been shown that the number of gear pairs in a geared mechanism cannot exceed the number of independent loops [10]; that is,

$$j2 \leq L \text{ . -------------------------- (9)}$$

By using eq.(6),(8),and(9),geared mechanisms can be designed and classified according to the number of dof ,L, and n.

In addition to satisfying the above structural characteristics, the following constraints are imposed [5]:

1. All links of an epicyclic gear train are capable of unlimited rotation.
2. For each gear pair, there exists a carrier, which keeps the center distance between the two meshing gears constant.

We study the effects of these two constraints on the structural characteristics of epicyclic gear trains. For all links to possess unlimited rotation, the prismatic joint is excluded from design consideration. Hence only revolute joints and gear pairs are allowed for structure synthesis of EGTs. For convenience, we use a thin edge to represent a revolute join (or turning pair) and a thick edge to stand for a gear pair. For this reason, thin edges are sometimes called turning-pair edges and thick edges are called geared edges.

Let j_g denote the number of gear pairs and j_t represent the number of revolute joints. It is clear that the total number of joints is given by equation (10).

$$j=j_t+j_g \text{-------------------- (10)}$$

Substituting Eq. (2) and (10) into Eq. (4), we obtain eq.(11).

$$F=3(n-1)-2j_t-j_g \text{ --------------- (11)}$$

The first constraint implies that there should not be any circuit formed exclusively by turning pairs. Otherwise, either the circuit will be locked or rotation of the links will be limited. The second constraint implies that all vertices should have at least one incident edge that represents a turning pair. Hence, we have

The sub graph obtained by removing all geared edges from the graph of an EGT is a tree.

Since a tree of v vertices contains v−1 edges, we further conclude that

$$j_t=n-1. \qquad (12)$$

Substituting Equations (10) and (12) into Equation (6), we obtain

$$j_g=L. \qquad (13)$$

Substituting Equations (12) and (13) into Equation (11) yields

$$L= n-1-F =j_t-F \text{ ------------------- (14)}$$

Eliminating j_t and j_g from Equations (10), (13),and (14), we get

$$j=F+2L \text{ ------------------ (15)}$$

Summarizing Equations (12), (13), and (14) in words, we have

"In epicyclic gear trains, the number of gear pairs is equal to the number of independent loops; the number of turning pairs is equal to the number of links diminished by one; and the number of degrees of freedom is equal to the difference between the number of turning pairs and the number of gear pairs".

We know that the minimum no. of joins on each link of a closed loop chain is 2. Therefore,

$d_i \geq 2$ -------------------------------- (16)

Using the fact that the no. of loops of which a vertex is a part is equal to its degree, and the maximum degree of a vertex is equal to the total no. of loops. Then

$L+1 \geq di$ ------------------------------- (17)

Combining eq. (16) and (17), we get

$L+1 \geq d_i \geq 2$ --------------- (18)

We can say that the minimum no. of joints on each link of a closed loop chain is 2 and maximum no. is limited by the total no. of loops. In other words, the degree of any vertex in the graph of an EGT lies between 2 and L+1.

In general, the graph of an EGT should not contain any circuit that is made up of only geared edges. Otherwise, the gear train may rely on special link length proportions to achieve mobility. In the case where geared edges form a loop, the number of edges must be even. Otherwise, the mechanism will not function properly.

Example -1
A two-dof differential gear train has been shown inFigure-8 in which gears3, 4, 5, and 6 forms a loop. The pitch diameter of gear 3 is equal to that of gear 5, and the pitch diameter of gear 4 is equal to that of gear 6. Otherwise, the mechanism will not function properly. Actually, we may consider either gear 4 or 6 as a redundant link. That is, removing either gear 4 or 6 from the mechanism does not affect the mobility of the mechanism. This is a typical fractionated mechanism in that links 2,3,4,5, and 6 form a one-dof gear train and the second degree of freedom comes from the fact that the gear train itself can rotate as a rigid body about the“a–a”axis.

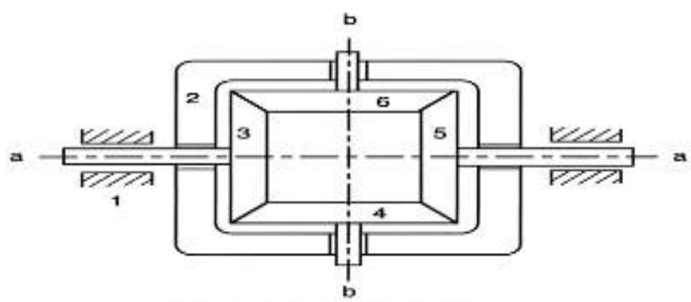

Figure- 8 A Differential gear train

CONCLUSION

In this paper, the author made the application of graph theory in the representation of P.G.T. It has been verified that in a geared mechanism, the no. of gear pairs cannot be more than the no. of independent loops. If there is unlimited rotation of all links in a geared mechanism then it is known as gear train. A P.G.T. is an assemblage of links and kinematic pairs. The study includes the functional schematic representation, structural representation, graph representation of P.G.Ts. The structural characteristics of EGTs are identified. It was shown that an n-link EGT contains $(n-1)$ turning pairs and $n-1-F$ gear pairs. The study is helpful for the U.G. /P.G. students, research scholars and designers in their early age of learning at the conceptual stage of design.

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